

MA11210 Differential Equations – Exercise Sheet 3

Please hand in solutions to the starred questions (1a, 1c, 1e, 2b, 2d, 4) via Blackboard as a single PDF file.

1. Find the general solutions of the following second order differential equations:

★ (a) $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 12y = 24x^2$ [6]

(b) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 2e^{-x}$

★ (c) $\frac{d^2y}{dx^2} + 4y = 10 \sin 3x$ [5]

(d) $\frac{d^2y}{dx^2} + 9y = 9 \sin 3x$

★ (e) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 10e^{3x}$ [8]

2. Find the solutions of the following differential equations that satisfy the given initial conditions:

(a) $y'' + 5y' + 6y = 30 \cos t, \quad y(0) = 4, \quad y'(0) = -1$

★ (b) $y'' - 2y' + 2y = 2t + 5e^{-t}, \quad y(0) = 0, \quad y'(0) = -1$ [6]

(c) $y'' + 4y = 10e^t - 4 \cos 2t, \quad y(0) = 5, \quad y'(0) = 6$

★ (d) $y'' + 4y' + 4y = 10 + 8t - 6te^{-2t}, \quad y(0) = 2, \quad y'(0) = 1$ [8]

3. Find the general solution of the following third order differential equation

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 10 \sin 3x - 5e^{-x}.$$

★ 4. Find the solution of the following fourth order differential equation, given that $y(0) = 3, y'(0) = 1, y''(0) = 2$ and $y'''(0) = -2$.

$$y^{(4)} - 4y'' = 8e^{2x} + 24x.$$

[7]