MA11210 Differential Equations – Exercise Sheet 2

Please hand in solutions to the starred questions (Q1 (all parts), Q2a, Q3a, Q3c(i), Q4b, Q4c) via Blackboard by 5pm on Monday, 2nd March.

⋆ 1. For each of the first order ordinary differential equations (a)-(e), select a method that can be used to solve the equation from the following list (*Note: in this question you are not required to solve the ODEs*):

- ODE is first order and linear - use an integrating factor.
- Solve by direct integration.
- Solve by the method of separable variables.
- ODE is homogeneous – making the substitution $y = vx$ will reduce the problem to separable form.
- Can be reduced to homogeneous form via the substitutions $x = x_0 + u$ and $y = y_0 + v$, where x_0 and y_0 are constants.

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + 6xy = 3x$

(b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y+2}{x-3}$

(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{18x^2+4}{3x^3+2x}$

(d) $(y^2+2x^2)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x^2 + 3xy$

(e) $x\dfrac{\mathrm{d}y}{\mathrm{d}x} + xy\cos(2x) - 7x^2 = 0.$

[*Hint: The flowchart on the module webpages may be useful*] [5]

2. Solve the following first order homogeneous differential equations, subject to the given boundary conditions:

⋆(a) $xy^3\dfrac{\mathrm{d}y}{\mathrm{d}x} = x^4 + y^4, \quad y(1) = 2$ [5]

(b) $x(2y+x)\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3y^2 + 2xy, \quad y(2) = 4$

3. Consider the first order differential equation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{ax+by+c}{lx+my+n}$$

where a, b, c, l, m and n are constants.

⋆(a) Suppose that $am - bl \neq 0$. Reduce the differential equation to a homogeneous first order differential equation by using the substitutions $x = u + x_0$ and $y = v + y_0$, where u and v are functions of x, and x_0 and y_0 are appropriately chosen constants dependent on a, b, c, l, m and n.
[*Hint: See lecture notes and/or module webpages!*]. [2]

(b) Suppose that $am = bl \neq 0$. Reduce the differential equation to a separable variable differential equation of the form $\frac{du}{dx} = F(u)$, where F is some function of u, by the substitution

$$u = lx + my + n.$$

[*Hint: First differentiate u with respect to x . . .*]

(c) Hence, find the general solutions of the following differential equations:

⋆i. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{x+y+2}{x-y+2}$. [8]

ii. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{6-2x-2y}{2x+2y+5}$.

4. Use an integrating factor to determine the general solutions of the following differential equations:

(a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + 3y = 6$

⋆(b) $\dfrac{\mathrm{d}x}{\mathrm{d}t} - \dfrac{2x}{t} = 2t^3 + 4t^2e^{4t}$ [6]

and the particular solutions of the following:

⋆(c) $\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = e^{-x}, \quad y(0) = 1$ [4]

(d) $x\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y = 10x^2, \quad y(1) = 3.$