MA11210 Differential Equations – Exercise Sheet 1

To be attempted before your tutorial on Friday 21st February
at which the questions will form the basis of a discussion.

1. Find the general solutions of the following exact differential equations:

   (i) $\dfrac{dy}{dx} = \sin 2x - e^{3x}$ [Hint: Direct integration]

   (ii) $\dfrac{dy}{dx} = \cot 4x$ [Hint: Use $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$]

   (iii) $\dfrac{dy}{dx} = \dfrac{5}{(x+3)(x-2)}$ [Hint: Use partial fractions]

   (iv) $\dfrac{dy}{dx} = \dfrac{x+2}{x(x+1)^2}$ [Hint: Use partial fractions]

   (v) $\dfrac{dy}{dx} = \dfrac{2}{9\cos^2 2x - \sin^2 2x}$ [Hint: Use $t = \tan 2x$]

   (vi) $\dfrac{dy}{dx} = x\ln[(x+1)^2] \qquad (x > -1)$ [Hint: Use integration by parts]

2. Write down the order and degree of each of the following differential equations and state whether or not they are linear.

   (i) $\dfrac{dy}{dx} + 3y = e^x$

   (ii) $\left(\dfrac{d^2y}{dx^2}\right)^4 + \dfrac{dy}{dx} = 3$

   (iii) $\dfrac{d^3y}{dx^3} + x\dfrac{dy}{dx} - 2x^2 = 0$

   (iv) $\dfrac{d^2y}{dx^2} + y^2 = 0$

   (v) $\dfrac{d^2y}{dx^2} - e^x\cos(x)\dfrac{dy}{dx} + 3\sin(x)y = e^{\sin x}$

   (vi) $\dfrac{d^3y}{dx^3} - \left(\dfrac{d^5y}{dx^5}\right)^4 - \dfrac{dy}{dx} = -x$

   (vii) $\dfrac{dy}{dx} + \cos x = 0$

   (viii) $\dfrac{dy}{dx} + \cos y = 0$

3. Use separation of variables to solve the following differential equations:

   (i) $\sin x\dfrac{dy}{dx} = y\cos x + \cos x$, given that $y(\frac{3\pi}{2}) = -2$.

   (ii) $x^2(y^2+1)\dfrac{dy}{dx} - y^2 = 0$, given that $y(2) = 1$.

   (iii) $(2x+3)\dfrac{dy}{dx} = \dfrac{1}{(y+2)(y-3)}$, given that $y(-2) = 1$.