MT11210 Hafaliadau Differol – Taflen Ymarfer 1 – Datrysiadau

1.(i) $\dfrac{dy}{dx} = \sin 2x - e^{3x}$

$$y = \int \sin 2x dx - \int e^{3x} dx + A \quad \text{(lle mae } A \text{ yn gysonyn mympwyol)}$$

$$\therefore \quad y = -\frac{1}{2}\cos 2x - \frac{1}{3}e^{3x} + A \quad \text{yw'r datrysiad cyffredinol.}$$

1.(ii) $\dfrac{dy}{dx} = \cot 4x$

$$y = \int \cot 4x \, dx + A = \int \frac{\cos 4x}{\sin 4x}\, dx + A \quad \text{(lle mae } A \text{ yn gysonyn mympwyol)}$$

$$\therefore \quad y = = \frac{1}{4}\ln|\sin 4x| + A \qquad \text{(gan fod } \int \frac{f'(x)}{f(x)}\, dx = \ln|f(x)|)$$

yw'r datrysiad cyffredinol.

1.(iii) $\dfrac{dy}{dx} = \dfrac{5}{(x+3)(x-2)}$

$$\text{Mae} \quad \frac{5}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2} \quad \text{ar gyfer rhyw gysonion } A, B$$

$$= \frac{A(x-2)+B(x+3)}{(x+3)(x-2)}$$

$$\text{Mae hafalu'r rhifiadur yn rhoi} \quad 5 = A(x-2) + B(x+3)$$

Rhowch $x = 2$: $\quad 5 = 5B \quad \Rightarrow B = 1.$
Rhowch $x = -3$: $\quad 5 = -5A \Rightarrow A = -1.$

$$\therefore \quad y = \int \frac{5}{(x+3)(x-2)} dx + C \quad \text{(lle mae } C \text{ yn gysonyn mympwyol)}$$

$$= \int \frac{1}{x-2} dx - \int \frac{1}{x+3} dx + C$$

$$= \ln|x-2| - \ln|x+3| + \ln D \quad (C = \ln D)$$

$$\therefore \quad y = \ln\left(D\left|\frac{x-2}{x+3}\right|\right) \quad \text{yw'r datrysiad cyffredinol.}$$

1.(iv) $\dfrac{dy}{dx} = \dfrac{x+2}{x(x+1)^2}$

$$\text{Mae} \quad \frac{x+2}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} \quad \text{ar gyfer rhyw gysonion } A, B, C$$

$$= \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$$

$$\text{Mae hafalu'r rhifiaduron yn rhoi} \quad x+2 = A(x+1)^2 + Bx(x+1) + Cx$$

Rhowch $x = -1$: $\quad 1 = -C \Rightarrow C = -1.$
Rhowch $x = 0$: $\quad 2 = A \quad \Rightarrow A = 2.$
Cymharu cyfernodau o $x^2 : 0 = A + B \Rightarrow B = -2.$

$$\therefore \quad y = \int \frac{2}{x}dx - \int \frac{2}{x+1}dx - \int \frac{1}{(x+1)^2}dx + D \quad \text{(lle mae } D \text{ yn gysonyn mympwyol)}$$
$$= 2\ln|x| - 2\ln|x+1| + \frac{1}{x+1} + D$$
$$\therefore \quad y = \ln\left(\frac{x}{x+1}\right)^2 + \frac{1}{x+1} + D \quad \text{yw'r datrysiad cyffredinol.}$$

1.(v) $\qquad \dfrac{dy}{dx} = \dfrac{2}{9\cos^2 2x - \sin^2 2x}$

Boed i $t = \tan 2x$ felly mae $\dfrac{dt}{dx} = 2\sec^2 2x$. Yna mae

$$y = \int \frac{2}{9\cos^2 2x - \sin^2 2x}\,dx$$
$$= \int \frac{2}{\cos^2(9 - \tan^2 2x)}\,dx$$
$$= \int \frac{2\sec^2 2x}{9 - \tan^2 2x}\,dx$$
$$= \int \frac{1}{9-t^2}\frac{dt}{dx}\,dx$$
$$= \int \frac{1}{9-t^2}\,dt$$

Mae $\dfrac{1}{9-t^2} = \dfrac{A}{3-t} + \dfrac{B}{3+t}$ ar gyfer rhyw gysonion A, B

$$= \frac{A(3+t) + B(3-t)}{9-t^2}$$

Mae hafalu'r rhifiaduron yn rhoi $1 = A(3+t) + B(3-t)$

Rhowch $t = 3$: $\quad 1 = 6A \Rightarrow 1 = 6A$
Rhowch $t = -3$: $\quad 1 = 6B \Rightarrow 1 = 6B$

Felly mae $y = \dfrac{1}{6}\displaystyle\int \frac{1}{3-t}\,dt + \frac{1}{6}\int \frac{1}{3+t}\,dt + C$ lle mae C yn gysonyn mympwyol

$$= -\frac{1}{6}\ln|3-t| + \frac{1}{6}\ln|3+t| + \ln D \quad (C = \ln D)$$
$$\therefore \quad y = \ln\left\{D\left|\frac{3+\tan 2x}{3-\tan 2x}\right|^{\frac{1}{6}}\right\} \quad \text{yw'r datrysiad cyffredinol.}$$

1.(vi) $\dfrac{dy}{dx} = x\ln(x+1)^2 \quad (x > -1)$

Felly mae $\dfrac{dy}{dx} = 2x\ln(x+1)$, gan fod $x+1 > 0$.

Integru fesul rhan: $$\begin{aligned} y &= (\ln(x+1))(x^2) - \int (x^2)\left(\frac{1}{x+1}\right) dx + A \quad (A \text{ cysonyn mympwyol}) \\ &= x^2\ln(x+1) - \int \left\{x - 1 + \frac{1}{x+1}\right\} dx + A \quad \text{trwy ffracsiynau rhannol} \\ &= x^2\ln(x+1) - \frac{1}{2}x^2 + x - \ln(x+1) + A \\ \therefore \quad y &= (x^2-1)\ln(x+1) - \frac{1}{2}x^2 + x + A \quad \text{yw'r datrysiad cyffredinol.} \end{aligned}$$

2. (i) Trefn un, gradd un, llinol.
(ii) Trefn dau, gradd pedwar, aflinol.
(iii) Trefn tri, gradd un, llinol.
(iv) Trefn dau, gradd un, aflinol.
(v) Trefn dau, gradd un, llinol.
(vi) Tefn pump, gradd pedwar, aflinol.
(vii) Trefn un, gradd un, llinol.
(viii) Trefn un, gradd un, aflinol.

3.(i) $\sin x\dfrac{dy}{dx} = y\cos x + \cos x$, o wybod fod $y(\frac{3\pi}{2}) = -2$.

Mae ad-drefnu yn rhoi $\sin x\frac{dy}{dx} = (y+1)\cos x$. Mae gwahanu'r newidynnau ac integru yn rhoi

$$\begin{aligned} \int \frac{1}{y+1}\, dy &= \int \cot x\, dx + A \quad (A \text{ cysonyn mympwyol}) \\ \ln|y+1| &= \ln|\sin x| + A \\ \text{Therefore, } y &= B\sin x - 1, \text{ lle mae } B \text{ yn gysonyn mympwyol, yw'r datrysiad cyffredinol.} \end{aligned}$$

Mae defnyddio'r amodau ffin yn rhoi:

$$-2 = y\left(\frac{3\pi}{2}\right) = B\sin\frac{3\pi}{2} - 1 = -B - 1 \Rightarrow B = 1$$

Felly, $y = \sin x - 1$ yw'r datrysiad penodol.

3.(ii) $x^2(y^2+1)\dfrac{dy}{dx} - y^2 = 0$, o wybod fod $y(2) = 1$.

Mae ad-drefnu yn rhoi $\dfrac{y^2+1}{y^2}\dfrac{dy}{dx} = \dfrac{1}{x^2}$. Mae integru yn rhoi:

$$\begin{aligned} \int \left\{1 + \frac{1}{y^2}\right\} dy &= \int \frac{1}{x^2}\, dx + A \quad \text{lle mae } A \text{ yn gysonyn mympwyol} \\ y - \frac{1}{y} &= -\frac{1}{x} + A \\ \therefore \quad y - \frac{1}{y} + \frac{1}{x} &= A \quad \text{yw'r datrysiad cyffredinol.} \end{aligned}$$

Mae defnyddio'r amod ffin $y(2) = 1$ yn rhoi:

$$A = 1 - \frac{1}{1} + \frac{1}{2} = \frac{1}{2}.$$

Felly, y datrysiad penodol yw $y - \frac{1}{y} + \frac{1}{x} = \frac{1}{2}$.

3.(iii) $(2x+3)\dfrac{dy}{dx} = \dfrac{1}{(y+2)(y-3)}$, o wybod fod $y(-2) = 1$.

Mae ad-drefnu yn rhoi, $(y+2)(y-3)\dfrac{dy}{dx} = \dfrac{1}{2x+3}$. Mae integru yn rhoi:

$$\begin{aligned}
\int (y+2)(y-3)\,dy &= \int \frac{1}{2x+3}\,dx + A \quad \text{lle mae } A \text{ yn gysonyn mywmpwyol} \\
\int \left\{y^2 - y - 6\right\}\,dy &= \frac{1}{2}\ln|2x+3| + A \\
\therefore \quad \frac{1}{3}y^3 - \frac{1}{2}y^2 - 6y &= \frac{1}{2}\ln|2x+3| + A \quad \text{yw'r datrysiad cyffredinol.}
\end{aligned}$$

Mae defnyddio'r amod ffin $y(-2) = 1$ yn rhoi:

$$\frac{1}{3} - \frac{1}{2} - 6 = \frac{1}{2}\ln|-1| + A \implies A = -\frac{37}{6}.$$

Felly, y datrysiad penodol yw

$$\frac{1}{3}y^3 - \frac{1}{2}y^2 - 6y = \frac{1}{2}\ln|2x+3| - \frac{37}{6}.$$